

Intrinsic Thickness of QCD Flux-Tubes

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Abstract

The effects of finite intrinsic thickness of QCD flux-tubes are explored using gauge/gravity duality under the assumptions that the position of the string in the fifth dimension is related to the intrinsic thickness of the QCD flux-tube, and the action of the five-dimensional fundamental string is the Nambu-Goto action. Under these assumptions the static $q\bar{q}$ potential is calculated in the large d approximation, where d is the number of transverse direction in the flat space where the gauge theory lives. The potential is found to be given by Arvis's formula corrected by an exponentially suppressed term, which can be thought of as arising from the fluctuations of the intrinsic thickness of the QCD flux-tube. This result suggests that when the effects of intrinsic thickness can be ignored, then the four-dimensional effective string theory is just the Nambu-Goto string with no additional marginal or irrelevant terms.

1 Introduction

In the past few years the lattice simulation of QCD flux-tubes has reached a level of precision where they can be unambiguously compared with the expectations from various effective string descriptions (two of the most recent investigations are [1, 2], and a lucid review with a history of numerical simulations of flux-tube using lattice gauge theory is given in [3].) These results show that the ground state energy of a long flux-tube matches very well with that obtained by Arvis [4] for the Nambu-Goto string in four flat dimensions. It was also pointed out in [2], that there are few excited states of a closed flux-tube that do show a large deviation from the Nambu-Goto prediction.

Excited states of open flux-tube have only been calculated in three dimensions [5] and do not show any unambiguous deviations from the predictions of the Nambu-Goto string, within the accuracy of the simulations.

The effective string theory description of QCD flux-tubes assumes that the length of the flux-tube, L , is much greater than any intrinsic thickness, l_w , of the flux-tube. Space-time history of such a flux-tube, in the absence of dynamical quarks, can be approximated by a two-dimensional surface or the world-sheet. The dynamics of this world-sheet is independent of the two-dimensional coordinate system used to describe it. A general reparametrization invariant action can be organised into relevant, marginal and irrelevant terms by dimensional analysis. The relevant term, which is proportional to the area of the world-sheet, is the Nambu-Goto action. Then there are marginal and irrelevant terms constructed from various powers of intrinsic and extrinsic curvatures (see for e.g. [6]).

In describing the dynamics of a QCD flux-tube using the Nambu-Goto action alone we expect two types of corrections: one that comes from the marginal and the irrelevant terms in the effective string action, and the other from the non-stringy degrees of freedom of a QCD flux-tube. Later being encapsulated in the notion of an intrinsic thickness of a flux-tube. The lattice results indicate that both types of correction to the static potential at sufficiently large distances are small. The absence of stringy corrections, that is the correction to the Nambu-Goto action from the marginal and the irrelevant terms, has been greatly elucidated in the recent theoretical investigations [7, 8, 9, 10, 11, 12, 13, 14]. The aim of the present investigation is to try and understand the nature of the correction due to the intrinsic thickness of the QCD flux-tube.

The notion of an intrinsic thickness of a flux-tube, though well motivated, is imprecise in QCD as we do not understand their dynamical origin. It is here that one expects that the conjectured gauge/gravity duality [15, 16, 17, 18] should allow us to delineate the effects of the intrinsic thickness of the flux-tube. A fundamental feature of the gauge/gravity duality is that the strings dual to the gauge theory live in (at least) five-dimensional curved space. These five-dimensional strings then should provide an *exact* description of the four-dimensional QCD flux-tubes. These fundamental strings have at least one additional degree of freedom as compared to the effective four-dimensional strings. It is tempting to associate this additional degree of freedom with the intrinsic thickness of a QCD flux-tube. This can be made more precise in the context of AdS/CFT correspondence [17, 19, 20], by looking at the size of the holographic projection of a five-dimensional object on to the four-dimensions.

AdS/CFT correspondence [21] can be regarded as an example of a more

general gauge/gravity duality [18] according to which all gauge theories, including QCD, have a dual description in terms of fundamental strings living in a higher-dimensional curved space-time. For QCD we do not know the precise geometry in which these putative five-dimensional strings live, but an approximate description which captures some of the properties of the QCD can be obtained by modifications of AdS_5 space. A particularly simple modification was used in [20] to understand the form factors of hadrons in terms of the intrinsically thick effective QCD strings arising from the holographic projection of the fundamental five-dimensional strings. We will use their modification to investigate the correction to the static potential due to the intrinsic thickness of the QCD flux-tube.

The outline of the paper is as follows: In sec. (2) the classical solution of the five dimensional string in the assumed confining geometry is described, and is interpreted as a static flux-tube of finite intrinsic thickness. In the same section the quadratic fluctuations about the classical solution are considered, and the mode corresponding to the fluctuation of the intrinsic thickness is found to be massive. Classical solution of a string in confining geometries and their quantum fluctuations have of course been studied since the very inception of the AdS/CFT conjecture (an incomplete list of references is [22, 23, 24, 25, 26, 27, 28]), our emphasize in this section is to relate the intrinsic thickness of the QCD flux-tube to the position of the string along the fifth direction. In sec.(3) we calculate the heavy quark potential in the large d approximation [29], where d is the number of transverse directions in the flat space where QCD is defined. The resulting potential is given by Arvis's formula [4] for the Nambu-Goto string corrected by a term which is exponentially suppressed by the factor of $\exp(-ML)$ where M^{-1} is a measure of the intrinsic thickness of the QCD flux-tube. This result is compared with the static potential obtained from an effective string action which has, in addition to the Nambu-Goto term, a marginal term, the so called rigidity term. We abstract a picture of QCD effective strings suggested by our result and state our conclusions in the final section (4).

2 Flux-tube from a Fundamental String

One way of stating the conjectured gauge/gravity duality is via the expectation value of a Wilson loop. The expectation value of a Wilson loop in a

four-dimensional Yang-Mills gauge theory is formally given by

$$\begin{aligned} W[\Gamma, A] &= \text{Tr} \hat{P} \exp \left\{ i \oint_{\Gamma} A \right\}, \\ \langle W[\Gamma] \rangle_{YM} &= \int [DA] \exp \{ -S_{YM}[A] \} W[\Gamma, A], \end{aligned} \quad (2.1)$$

where Γ is a non-intersecting closed loop in four-dimensions, A represents the matrix valued vector potential of the $SU(N)$ Yang-Mills theory, which have been path-ordered along the curve Γ using the path ordering operator \hat{P} , and $S_{YM}[A]$ is the Euclidean Yang-Mills action for the gauge fields. The gauge/gravity duality then implies that this expectation value is also given by a weighted sum over all the surfaces whose boundary is the curve Γ which lives in the flat four-dimensional space, while the surfaces themselves live in a curved five-dimensional space. Symbolically

$$\langle W[\Gamma] \rangle_{YM} = \int [DX] \exp \left\{ -T_o \int d^2\sigma \sqrt{\gamma[X]} \right\}, \quad (2.2)$$

where

$$X^m = X^m(\sigma^0, \sigma^1), \quad m = 1, \dots, 5 \quad (2.3)$$

is the string surface parametrized by the parameters $\{\sigma^0, \sigma^1\}$, γ is the determinant of the induced metric

$$\gamma_{ab} = g_{mn} \frac{\partial X^m}{\partial \sigma^a} \frac{\partial X^n}{\partial \sigma^b}, \quad (2.4)$$

and T_o is the string-tension. The curved five-dimensional space is described by the metric g_{mn} , which following [20] will be taken to be of the following form

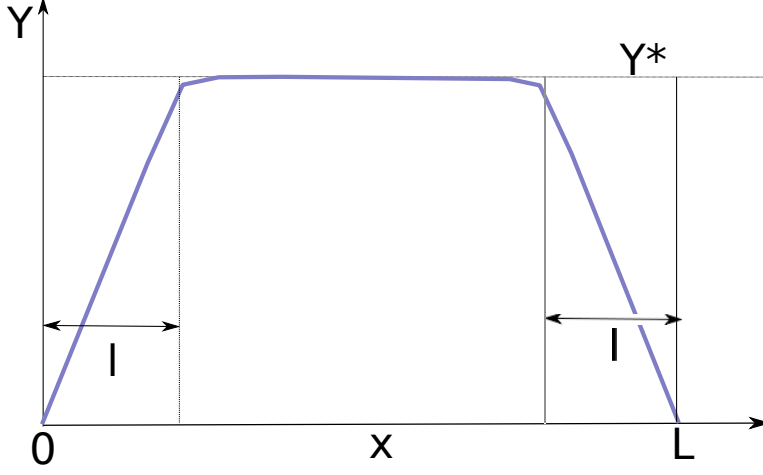
$$\begin{aligned} ds^2 = g_{mn} dx^m dx^n &= F(x_5) (dx_4^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_5^2) \\ &= F(y) (dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dy^2), \end{aligned} \quad (2.5)$$

where the function $F(y)$ is assumed to have a minimum at $y = y^*$ and for $y \rightarrow 0$ it approaches the AdS_5 limit

$$F(y) \approx \frac{R^2}{y^2}. \quad (2.6)$$

Consider now a Wilson loop made of the world-lines of a quark at the origin and an anti-quark placed along the x_1 axis at a coordinate distance of L . Working in the static gauge

$$\sigma^0 = \tau = X_4 = t; \quad \sigma^1 = \sigma = X_1 = x, \quad (2.7)$$

Fig. 1: $Y_c(x)$

the classical world-sheet, or the minimal surface, is of the form:

$$X_c = (\tau, \sigma, 0, 0, Y_c(\sigma)). \quad (2.8)$$

$Y_c(\sigma)$, which is a geodesic in the five-dimensional curved space, satisfies the equation

$$\frac{\partial}{\partial \sigma} \left(\frac{F[Y]Y'}{\sqrt{(1+Y'^2)}} \right) = F'[Y] \sqrt{(1+Y'^2)}. \quad (2.9)$$

With the assumed form of $F[y]$, qualitatively $Y_c(\sigma)$ looks like the curve shown in Fig.(1). In particular one can approximate

$$Y_c(x) = Y^* \quad (l \leq x \leq L-l). \quad (2.10)$$

The parameter Y^* and l will depend on the precise form of the confining metric. The corresponding classical action is then given by

$$\begin{aligned} S[Y_c] &= T_0 \int_{-\infty}^{\infty} dt \int_0^l dx F[Y_c] (1 + Y_c'^2)^{\frac{1}{2}} \\ &+ T_0 F[Y^*] \int_{-\infty}^{\infty} dt \int_l^{L-l} dx \\ &+ T_0 \int_{-\infty}^{\infty} dt \int_{L-l}^L dx F[Y_c] (1 + Y_c'^2)^{\frac{1}{2}}. \end{aligned} \quad (2.11)$$

Let us define a position dependent tension,

$$T(x) = \begin{cases} T_0 F[Y_c](1 + Y'^2)^{1/2} & 0 \leq x \leq l \\ T_0 F[Y^*] & l < x < L - l \\ T_0 F[Y_c](1 + Y'^2)^{1/2} & L - l \leq x \leq L \end{cases} \quad (2.12)$$

and write the classical action as

$$S_c = \int_{-\infty}^{\infty} dt \int_0^L dx T(x). \quad (2.13)$$

The above action can be thought of as an action for a string along the x axis in four-dimensions, but with a position dependent string tension. If one considers a flux-tube like the Nielsen-Olesen vortex line [30] then one finds that the string tension is related to the size of the flux-tube by

$$T \sim \frac{1}{\lambda^2}, \quad (2.14)$$

where λ characterizes the region beyond which the magnetic field is essentially zero. Therefore a suggestive interpretation of (2.13) is that it represents a flux-tube of varying intrinsic thickness. The assumed form of $T(x)$, (2.12), then implies that the classical flux-tube has a constant thickness in the region $l < x < L - l$, while the intrinsic thickness vanishes at the end-points as shown in Fig. 2. Also, note that the part where $T(x)$ is changing is the part where $Y_c(x)$ is changing, suggesting that it is the Y coordinate of the string that is related to the intrinsic thickness of the flux-tube in concordance with the argument of [20]. In the present investigation, we will consider the limit $l/L \ll 1$ for which (2.13) can be approximated by an action for a static-string in four-dimensions with the constant string tension given by $T_0 F[Y^*]$.

Next, let us consider the fluctuations around the classical string configuration (2.8). Again, working in the static gauge these fluctuations can be written as

$$\begin{aligned} X^m &= (t, x, X_2(t, x), X_3(t, x), Y_c(x) + \phi_y(t, x)) \\ &= \left(t, x, \vec{\phi}_T, Y_c(x) + \phi_y(t, x) \right), \\ \vec{\phi}_T &= (\phi_1(t, x), \phi_2(t, x)) = (X_2(t, x), X_3(t, x)) \end{aligned} \quad (2.15)$$

where $\vec{\phi}$ are the transverse fluctuations, while ϕ_y represent the fluctuation along the fifth-dimension. From the point of view of the QCD flux-tube, ϕ_y represents the fluctuation in the intrinsic thickness.

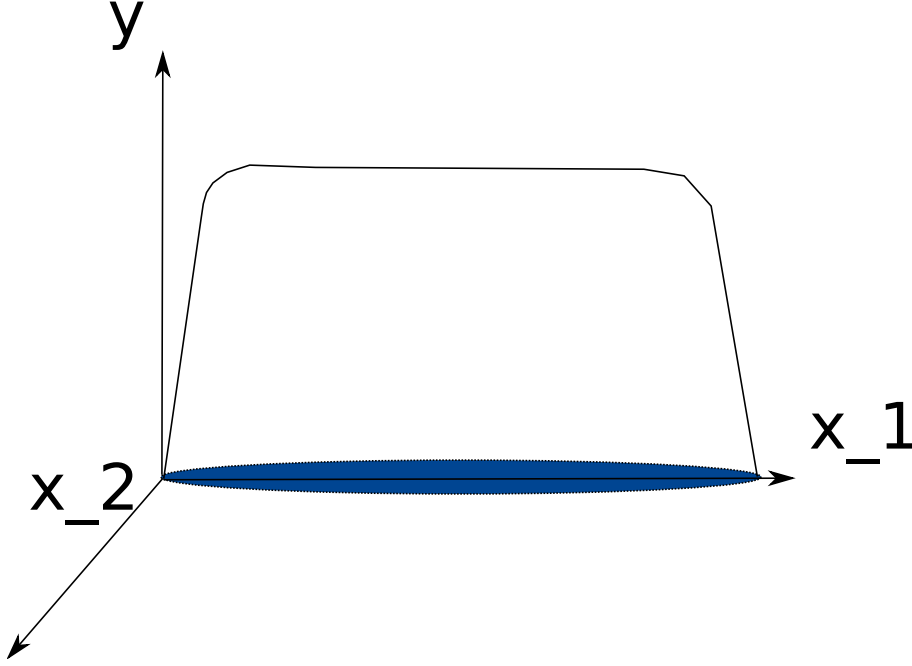


Fig. 2: Flux-tube from a string

The leading correction to the linear term in the static potential, the Lüscher term

$$V_L = -\frac{\pi}{24} \frac{1}{L} n_0, \quad (2.16)$$

depends on the number of massless modes, n_0 . Therefore it is important to determine whether the additional mode of the five dimensional fundamental string is massive or massless. To check this, we need to consider only terms which are quadratic in the fluctuating coordinates. To this order one finds

$$S[\vec{\phi}, \phi_y] = S_L + S_l, \quad (2.17)$$

where S_l is the only part of the action that depends on the factor l introduced in (2.10). Let us first focus on S_L which takes the following form

$$\begin{aligned} S_L = & T_0 F[Y^*] \int dt \int_0^L dx \\ & + T_0 F[Y^*] \int dt \int_0^L dx \frac{1}{2} \left((\partial_t \vec{\phi}_T)^2 + (\partial_x \vec{\phi}_T)^2 \right) \\ & + T_0 F[Y^*] \int dt \int_0^L dx \frac{1}{2} \left((\partial_t \phi_y)^2 + (\partial_x \phi_y)^2 + M^2 \phi_y^2 \right), \end{aligned} \quad (2.18)$$

and

$$M^2 = \frac{1}{F[Y^*]} \frac{d^2 F[Y^*]}{dY^2}. \quad (2.19)$$

If we substitute S_L in the expression for the expectation value of the Wilson loop, (2.2), then we readily obtain, in the limit $ML \gg 1$, an expression for the static potential¹

$$V(L) = T_0 F[Y^*] L - \frac{\pi}{12} \cdot \frac{1}{L} - \frac{1}{4\sqrt{\pi}L} \sqrt{ML} \exp(-2ML). \quad (2.20)$$

The static potential has, apart from the linear and the Lüscher term, an additional exponentially suppressed contribution from the massive mode ϕ_y . The mass of the ϕ_y mode comes from our assumption that $F[Y]$, which defines the confining metric (2.5), has a minimum at $Y = Y^*$. From dimensional considerations, one expects that M should be of the order of the lightest glue-ball mass which according to the lattice simulations is around 1 GeV.

The above considerations are only true for the case where $L \gg l$ (see Figure(1)). There are corrections to the above result for the static potential coming from S_l , but for confining geometries the ratio l/L tends to zero as L increases [28]. In the rest of this investigation we will ignore the contribution from S_l , except for some comments in the final section.

3 Static Potential due to a Flux-Tube

The gauge/gravity duality should inform us about the form of the effective string theory that describes QCD flux-tubes, and any correction to it due to the intrinsic thickness of the flux-tube. In the previous section we argued that a classical fundamental string in five-dimensions induces a flux-tube with finite intrinsic thickness in four-dimensions. Such a classical configuration leads to a linear static potential, which is corrected by the quantum fluctuation (2.20). Numerical simulations show that the static potential is described not just by a linear term plus the Lüscher term, but there are additional corrections which are best described by the Arvis's formula [4]. Therefore there is a need to improve open the approximation that led to (2.20).

According to our assumption (2.2), in the static gauge, the expectation value of a rectangular Wilson loop of size $T \times L$ can be written as

$$\langle W \rangle_{YM} = \int [d\vec{\phi}] \exp \left\{ -T_0 \int d^2\sigma F[Y] \det[\delta_{ab} + \partial_a \vec{\phi} \cdot \partial_b \vec{\phi}]^{\frac{1}{2}} \right\}, \quad (3.1)$$

¹ The exact expression for the contribution of the massive mode to the static potential is (see for e.g. [31]) $-\frac{1}{4\pi L} \int_{ML}^{\infty} \frac{\sqrt{y^2 - 4(ML)^2}}{\exp(y) - 1} dy$.

where $\vec{\phi}$ represent the fluctuations about the classical solution,

$$\begin{aligned} X(\sigma) &= \left\{ t, x, \vec{\phi}_T, Y_c(x) + \phi_y(t, x) \right\}, \\ \vec{\phi} &= \left\{ \vec{\phi}_T, \phi_y \right\}. \end{aligned} \quad (3.2)$$

As we saw in the previous section, the fluctuations $\vec{\phi}_T$ are mass-less, while the mode ϕ_y is massive. With this in mind we will approximate the warp factor $F(y)$ appearing in the metric 2.5 as

$$F[Y] \approx F^* + \frac{1}{2} \left(\frac{d^2 F}{dY^2} \right)_{Y^*} \phi_Y^2 = F^* \left(1 + \frac{1}{2} M^2 \phi_Y^2 \right), \quad (3.3)$$

and write the five-dimensional Nambu-Goto action as

$$S_{NG} = \frac{1}{l_s^2} \int d^2 \sigma \left(1 + \frac{1}{2} M^2 \phi_Y^2 \right) \det[\delta_{ab} + \partial_a \vec{\phi} \cdot \partial_b \vec{\phi}]^{\frac{1}{2}}, \quad (3.4)$$

with

$$\frac{1}{l_s^2} = T_0 F^*. \quad (3.5)$$

In this approximation, the expectation value of the Wilson-loop is given by

$$\langle W \rangle_{YM} = \int [d\vec{\phi}] \exp \left\{ -\frac{1}{l_s^2} \int d^2 \sigma \left(1 + \frac{1}{2} M^2 \phi_Y^2 \right) \det[\delta_{ab} + \partial_a \vec{\phi} \cdot \partial_b \vec{\phi}]^{\frac{1}{2}} \right\}. \quad (3.6)$$

Since $\vec{\phi}_T$ represents mass-less modes we would like to evaluate the above functional integral in a manner which treats them non-perturbatively. For this purpose we will employ a large d expansion for $\vec{\phi}_T$ [29]. First we introduce an auxiliary field

$$g_{ab} = \partial_a \vec{\phi} \cdot \partial_b \vec{\phi} \quad (3.7)$$

and write the expectation value of the Wilson loop as

$$\begin{aligned} \langle W \rangle_{YM} &= \int [d\vec{\phi}] [dg] \delta(g_{ab}(\sigma) - \partial_a \vec{\phi} \cdot \partial_b \vec{\phi}) \\ &\quad \times \exp \left\{ -\frac{1}{l_s^2} \int d^2 \sigma \left(1 + \frac{1}{2} M^2 \phi_Y^2 \right) \det[\delta_{ab} + g_{ab}]^{\frac{1}{2}} \right\}, \end{aligned} \quad (3.8)$$

and then write the delta-function in terms of a Lagrangian-Multiplier field, N_{ab} , to obtain

$$\langle W \rangle_{YM} = \int [d\vec{\phi}] [dg] [dN] \exp \left\{ -S[\vec{\phi}, g, N] \right\}, \quad (3.9)$$

with the action given by

$$S = \frac{1}{l_s^2} \int d^2\sigma \left\{ \left(1 + \frac{1}{2} M^2 \phi_Y^2\right) \det[\delta_{ab} + g_{ab}]^{\frac{1}{2}} + \frac{1}{2} N^{ab} (\partial_a \vec{\phi} \cdot \partial_b \vec{\phi} - g_{ab}) \right\}. \quad (3.10)$$

Since $\vec{\phi}$ appears quadratically it can be integrated to obtain

$$S_{eff}[N, g] = \frac{1}{l_s^2} \int d^2\sigma \left\{ A - \frac{1}{2} N^{ab} g_{ab} \right\} + \frac{d}{2} \text{Tr} \log (-\partial_a N^{ab} \partial_b) + \frac{1}{2} \text{Tr} \log (-\partial_a N^{ab} \partial_b + A M^2), \quad (3.11)$$

where

$$A = \det [\delta_{ab} + g_{ab}]^{\frac{1}{2}}. \quad (3.12)$$

The fields N_{ab} and g_{ab} are to be determined by minimizing S_{eff} . Taking a clue from (2.20), which implies that in the quadratic approximation both N and g are identity matrix (see also [29]), we assume that N and g are independent of (σ_0, σ_1) and are diagonal matrices

$$\begin{aligned} N &= \begin{bmatrix} N_0 & 0 \\ 0 & N_1 \end{bmatrix}, \\ g &= \begin{bmatrix} g_0 & 0 \\ 0 & g_1 \end{bmatrix}, \\ A &= \sqrt{(1 + g_0)(1 + g_1)}. \end{aligned} \quad (3.13)$$

Using this ansatz and after calculating the functional traces in the limit $ML \gg 1$ one obtains

$$\begin{aligned} S_{eff}[N, g] &= \frac{TL}{l_s^2} \left(A - \frac{1}{2} (N_0 g_0 + N_1 g_1) \right) - d\pi T \left(\frac{N_1}{N_0} \right)^{\frac{1}{2}} \frac{1}{24L} \\ &\quad - T \left(\frac{N_1}{N_0} \right)^{\frac{1}{2}} \frac{1}{4\sqrt{\pi}L} \left(\left(\frac{A}{N_1} \right)^{\frac{1}{2}} ML \right)^{\frac{1}{2}} \exp \left[-2 \left(\frac{A}{N_1} \right)^{\frac{1}{2}} ML \right]. \end{aligned} \quad (3.14)$$

To solve for N and g we write

$$S_{eff}[N, g] = S_0[N, g] + S_1[N, g], \quad (3.15)$$

where

$$\begin{aligned} S_0[N, g] &= \left(\frac{\pi T}{24L} \right) \frac{d}{\lambda} \left((1 + g_0)^{\frac{1}{2}} (1 + g_1)^{\frac{1}{2}} - \lambda \left(\frac{N_1}{N_0} \right)^{\frac{1}{2}} \right), \\ S_1[N, g] &= -\frac{T}{4\sqrt{\pi}L} \left(\frac{N_1}{N_0} \right)^{\frac{1}{2}} \left(\sqrt{BML} \exp \{-2BML\} \right), \end{aligned} \quad (3.16)$$

with

$$\begin{aligned}\lambda &= \frac{\pi d}{24} \left(\frac{l_s}{L} \right)^2, \\ B &= \left(\frac{A}{N_1} \right)^{\frac{1}{2}}.\end{aligned}\tag{3.17}$$

In the large d limit, obtained by keeping λ fixed, we can solve for N and g by minimizing $S_0[N, g]$ alone, which is same as the action considered in [29] and one obtains

$$\begin{aligned}\bar{N}_0 &= (1 - 2\lambda)^{\frac{1}{2}}, \\ \bar{N}_1 &= \frac{1}{(1 - 2\lambda)^{\frac{1}{2}}}, \\ \bar{g}_0 &= \frac{\lambda}{(1 - 2\lambda)}, \\ \bar{g}_1 &= -\lambda.\end{aligned}\tag{3.18}$$

Substituting the above solution in $S_{eff}[N, g]$ one obtains the static potential as:

$$V[L] = \frac{L}{l_s^2} \left(1 - \frac{L_c^2}{L^2} \right)^{\frac{1}{2}} - \frac{1}{4\sqrt{\pi}L} C[L] \sqrt{ML} \exp \left\{ -2 \left(1 - \frac{L_c^2}{2L^2} \right)^{\frac{1}{2}} ML \right\},\tag{3.19}$$

where

$$\begin{aligned}L_c^2 &= \frac{\pi d l_s^2}{12}, \\ \lambda &= \frac{L_c^2}{2L^2}, \\ C[L] &= \frac{(1 - \lambda)^{\frac{1}{4}}}{(1 - 2\lambda)^{\frac{1}{2}}}.\end{aligned}\tag{3.20}$$

The first term in (3.19) is the Arvis's formula for the static potential due to a Nambu-Goto string in $D = d + 2$ flat dimensional space [4] and was obtained originally in the large d expansion by Alvarez [29]. The second term in (3.19) arises from the fluctuations in the intrinsic thickness of the flux-tube. It is pertinent to note that this term is non-analytic in M^{-1} . Also, as expected, for small values of λ this contribution coincides with the second term of (2.20).

It is natural to ask, what kind of effective string theory in four-dimension can reproduce (3.19)? With this in mind, consider an effective string theory which is reparametrization invariant and includes all the relevant and marginal terms²:

$$S_{rigid} = M_0^2 \int d^2\sigma \sqrt{\gamma} + \frac{1}{2e^2} \int d^2\sigma \sqrt{\gamma} (\Delta(\gamma) X^\mu)^2, \quad (3.21)$$

this describes the so called rigid strings in four dimensions [32, 33, 34, 35, 36], where γ is the determinant of the induced metric, $\Delta(\gamma)$ is the corresponding Laplacian, M_0^2 is the bare string tension, and e is the dimensionless coupling constant for the rigidity term. The static potential corresponding to this action has been calculated in the large d expansion [37], and for large L it is given by

$$V_{rigid} = M^2 L - \frac{\pi d}{24} \frac{1}{L} - \frac{1}{8} \left(\frac{\pi d}{12} \right)^2 \frac{1}{M^2 L^3} + \frac{1}{8} \left(\frac{\pi d}{12} \right)^2 \left(\frac{3\pi}{10} \right) \frac{1}{e M^3 L^4} + \dots \quad (3.22)$$

Comparing (3.19) and (3.22), we note that even after ignoring the exponential term coming from the intrinsic thickness, the two differ at $O(1/L^4)$. Thus for the confining five dimensional geometry that we have assumed, there is no marginal term induced in the effective string theory in four dimensions. For a different confining geometry this might not be the case as suggested in [25]. This shows that precise lattice simulations can inform us about the conjectured fundamental string theory that is dual to QCD.

4 Conclusions

The gauge/gravity duality offers the possibility of illuminating the success and failures of the Nambu-Goto string in describing QCD flux-tubes. This possibility is of course tempered by the fact that we do not know the precise string theory in five dimensions that corresponds to QCD in four dimensions. In the absence of that, one can explore physically motivated five-dimensional string theories that captures some essential features, like confinement.

The main result of our calculations, (3.19), with in the approximations considered, has three salient features. Firstly, as far as the contribution to the static potential is concerned, the massless modes corresponding to the transverse fluctuations in four dimension, decouple from the massive mode arising from the fluctuation in the fifth dimension. Secondly, the contribution of the massless modes to the static potential is identical to that for the

² There is one more marginal term in four dimensions which is topological in nature but is not relevant for our discussion [6].

Nambu-Goto string in four-dimensions. Finally, the contribution of the massive mode, which corresponds to the fluctuations in the intrinsic thickness, are exponentially suppressed and are non-analytic in the intrinsic thickness of the flux-tube. In obtaining (3.19) we neglected the contribution from the fluctuation of the string in the region adjacent to the boundary, characterized by length l in Fig.(1), concentrating on the flux-tubes for which $L \gg l$. The physics in these regions is governed by the detailed form of $F(y)$ in (2.5). Since the Y coordinate of the string changes rapidly near the boundary, one would expect that effects of intrinsic thickness should be important, and they could couple with the massless modes. A possible hint of this is seen in the interaction of the spin of the quark and the anti-quark with the fluctuations of the flux-tube connecting them [38, 39]. Perhaps most importantly it is for $L \sim l$ that a realistic dual string description should reproduce the effects of Asymptotic freedom, and the static potential obtained through gauge/gravity duality should match the perturbatively results.

Our analysis suggests that long, $L \gg l$, open flux-tubes connecting quark with an anti-quark, are described just by the Nambu-Goto action with no contribution from a marginal rigidity term or any finite number of irrelevant terms. Therefore it will be very useful to have numerical results for the excited states of open flux-tube, and to see how well they match with the prediction of the Nambu-Goto string (excited states of open string in three dimension have been explored in [5].) If the suggested picture is correct then, for $L \gg l$, the only deviation from the Nambu-Goto string should come from the excitations of the massive modes related to the intrinsic thickness, and from their possible coupling with the massless modes. For closed flux-tube it has been pointed out in [2] that there are definite deviations from the Nambu-Goto string for the excited states, which may be a hint of the non-stringy modes related to the intrinsic thickness [3].

In our analysis, the origin of the Nambu-Goto term in the effective four dimensional string theory can be traced to our assumption that the fundamental string in five-dimension is described by the Nambu-Goto action alone, and to the specific curved geometry used to describe the confinement. In particular, the assumed form of the warp factor in the fifth dimension was responsible for making the mode associated with the fluctuation of the intrinsic thickness of the QCD flux-tube massive. A different kind of confining background could in principle induce an effective string description in four dimension which has in addition to Nambu-Goto term a marginal rigidity term and possibly irrelevant terms [25]. This shows, as we remarked at the end of sec.(3), that precise lattice measurement of static potential can provide us with a unique window into the geometry of the conjectured five-dimensional strings which are dual to QCD.

A fundamental bosonic string in five dimension by itself is probably not a consistent quantum mechanical system and a consistent description would correspondingly require additional bosonic and fermionic variables. Further, the background geometry in which these fundamental strings live should reproduce not only confinement but also the asymptotic freedom of QCD. Therefore our use of the gauge/gravity duality is at best an approximation. The success of the four-dimensional Nambu-Goto string in describing QCD flux-tube indicates that this approximation does capture some of the long distance physics of QCD. One of the attractive feature of our use of gauge/gravity duality is that it associates the extra degree of freedom of a five dimensional string with the intrinsic thickness of the QCD flux-tube. It will be very interesting to see if one can similarly relate the extra bosonic and fermionic modes required by a fundamental superstring with some degrees of freedom of QCD flux-tube.

Acknowledgements

I would like to thank Gunnar Bali, N. D. Hari Dass, and Rajesh Gopakumar for very useful discussions. I am particularly grateful to N. D. Hari Dass for his detailed comments on a preliminary version of this paper. I would also like to thank Andreas Athenodorou, Barak Bringoltz, and Mike Teper for a discussion on their results. This work was started while I was visiting the Institute of Theoretical Physics, at the University of Regensburg. I am grateful to the members of the Institute for their hospitality, particularly to Gunnar Bali for his invitation to visit the University of Regensburg, and DAAD for their financial support. I have equally benefited from my visit to HRI, Allahabad, and would like to thank Satchi Naik for his kind invitation.

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